

CS109 Section 1 Solutions

Problem 1

Let C be the event that the student knows geometry. Let S be the event that the student solves the first geometry question correctly. Then $P(C) = q$, $P(C^c) = 1 - q$, $P(S|C) = p_1$, and $P(S|C^c) = p_2$. The value we need to solve for is $P(C|S)$. We use Bayes Rule:

$$\begin{aligned}P(C|S) &= \frac{P(S|C)P(C)}{P(S)} \\P(C|S) &= \frac{P(S|C)P(C)}{P(S|C)P(C) + P(S|C^c)P(C^c)} \\P(C|S) &= \frac{p_1q}{p_1q + p_2(1 - q)}\end{aligned}$$

Problem 2

Let S be the event the student solves the first question correctly.

A = the event the student is in the Beginner level

B = the event the student is in the Intermediate level

C = the event the student is in the Expert level

D = the event the student is in the Euclid level

Then $P(A) = b_1$, $P(B) = b_2$, $P(C) = b_3$, $P(D) = b_4$. And $P(S|A) = d_1$, $P(S|B) = d_2$, $P(S|C) = d_3$, $P(S|D) = d_4$. We want to solve for $P(A|S)$, $P(B|S)$, $P(C|S)$, $P(D|S)$. Here we show the example for $P(A|S)$. We use Bayes Rule:

$$\begin{aligned}P(A|S) &= \frac{P(S|A)P(A)}{P(S)} \\P(A|S) &= \frac{P(S|A)P(A)}{P(SA) + P(SB) + P(SC) + P(SD)} \\P(A|S) &= \frac{P(S|A)P(A)}{P(S|A)P(A) + P(S|B)P(B) + P(S|C)P(C) + P(S|D)P(D)} \\P(A|S) &= \frac{d_1b_1}{d_1b_1 + d_2b_2 + d_3b_3 + d_4b_4}\end{aligned}$$

Note that we get the second equation by the law of total probability.